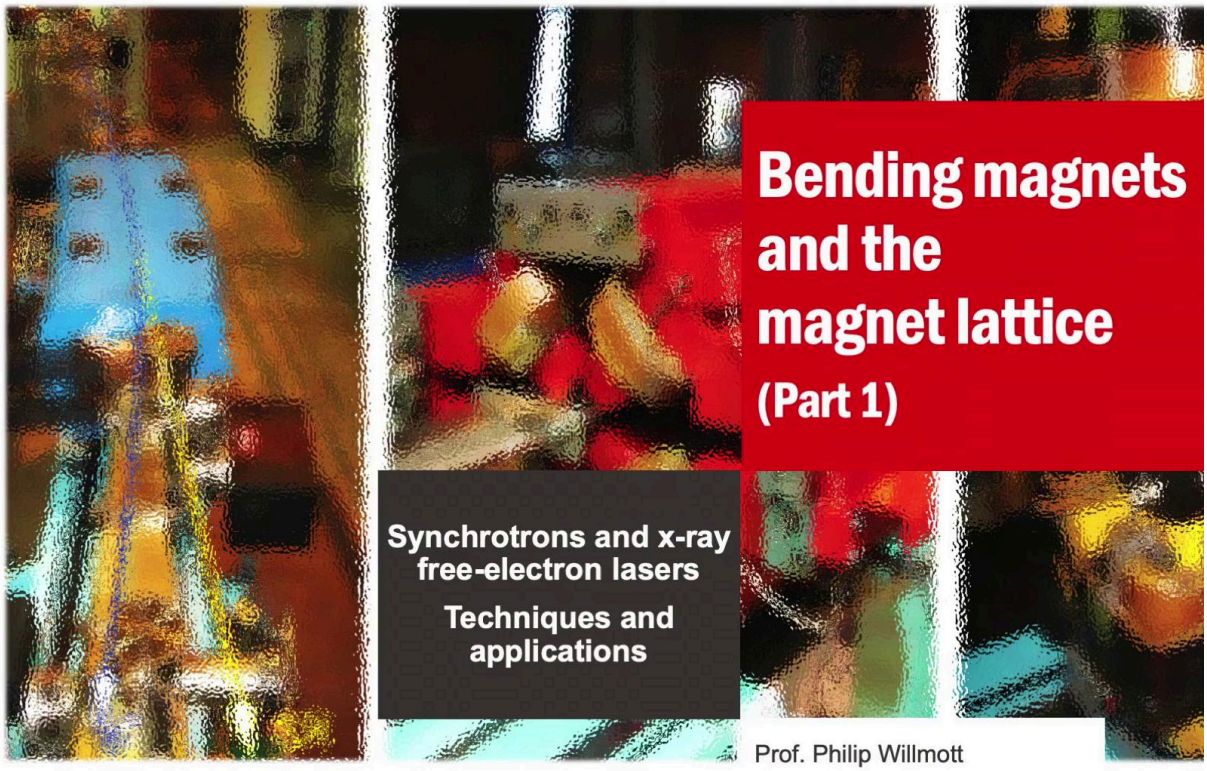
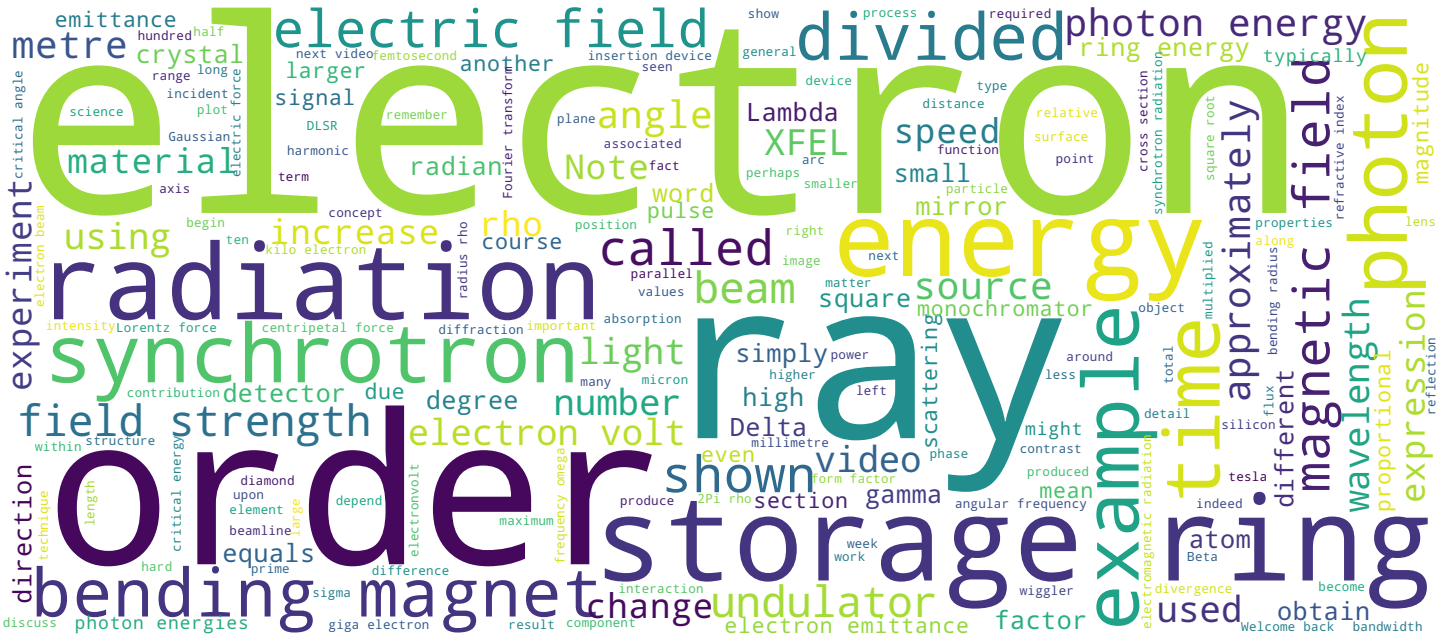
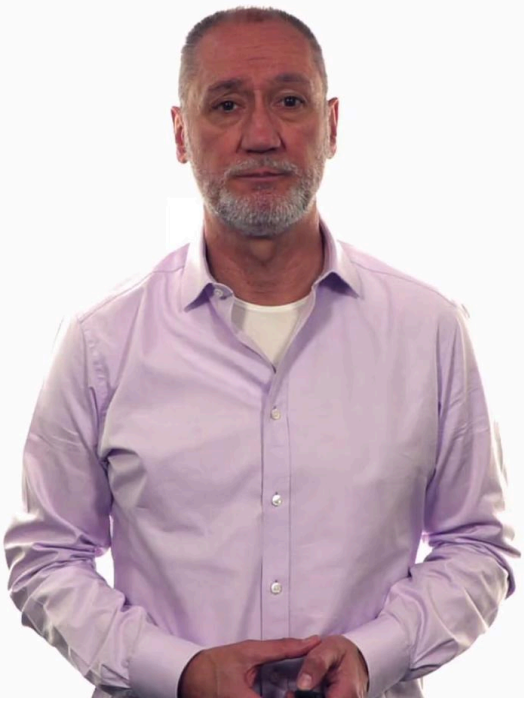


École polytechnique fédérale de Lausanne


electron
radiation
synchrotron
order
storage ring
bending magnet
electric field
divided
photon energy
energy
photon
ray
example
time
magnetic field
wavelength
undulator
shown
video
material
angle
XFEL
speed
Note
called
beam
source
light
field strength
electron volt
increase
using
experiment
emittance
metre
crystal
signal
larger
another
Lambda
small
mirror
monochromator
scattering
approximately
different
expression
proportional
atom
used
obtain
change
factor
electron emittance
photon energies
direction
equals
diamond
beamline
detector
due
degree
number
high
Delta
simply
rho
course
pulse
word
radian
Fourier transform
insertion device
synchrotron radiation
cross section
refractive index
magnitude
lens
reflection
square root
bending radius
multiplied
properties
along
object
detail
silicon
tesla
indeed
become
Welcome back
bandwidth
divergence
wiggler
angular frequency
contrast
mean
produced
around
less
phase
gamma
section
produce
2Pi rho
electromagnetic radiation
electronvolt
frequency omega
maximum
difference
sigma
giga electron
result
component
interaction
week
work
Beta
prime
hard
technique
length
critical energy
critical angle
electric force
electron beam
intensity
Lorentz force
centripetal force
diffraction
important
absorption
contribution
many
micron
parallel
values
position
radius rho
begin
ten
kilo electron
concept
axis
term
harmonic
DLSR
Gaussian
plot
incident
range
long
hundred
half
emittance
metre
science
remember
plane
distance
type
device
general
seen
associated
fact
perhaps
smaller
arc
function
point
relative
surface
particle
right
image
next
matter
square
power
higher
left
millimetre
even
video
form factor
many
micron
within
structure
discuss
upon
element
large
depend
prime
hard
equals
beamline
sigma
difference
interaction
week
work
wiggler
divergence
become
Welcome back
bandwidth
Beta
atom
used
obtain
factor
electron emittance
photon energies
giga electron
result
component
produce
2Pi rho
section
gamma
angular frequency
contrast
mean
produced
around
less
phase
tesla
indeed
silicon
detail
object
total
flux
bending radius
multiplied
properties
along
lens
reflection
square root
cross section
synchrotron radiation
refractive index
magnitude
electromagnetic radiation
electronvolt
frequency omega
maximum
difference
sigma
giga electron
result
component
interaction
week
work
wiggler
divergence
become
Welcome back
bandwidth
Beta
atom
used
obtain
factor
electron emittance
photon energies
direction
equals
diamond
beamline
detector
due
degree
number
high
Delta
simply
rho
course
pulse
word
radian
Fourier transform
insertion device
synchrotron radiation
cross section
refractive index
magnitude
lens
reflection
square root
bending radius
multiplied
properties
along
object
detail
silicon
tesla
indeed
become
Welcome back
bandwidth
divergence
wiggler
angular frequency
contrast
mean
produced
around
less
phase
gamma
section
produce
2Pi rho

Contents and objectives of this video



- Equating Lorentz and centripetal forces
- Bending magnets
  - Bending radius
  - Typical B-fields
  - The angular frequency
  - Bending-magnet spectra
  - The critical frequency

Welcome back to the third section of week three of this six-week course on Synchrotrons and exhales. In these videos, we'll consider the most important elements within a storage ring that are required to keep the electrons on a stable and closed orbit. We reserve discussion of insertion devices to next week's videos. In this first video, we will calculate the order of magnitude strength of the magnetic fields required to impart the necessary centripetal forces on the electrons and other important parameters associated with bending magnet dipoles.

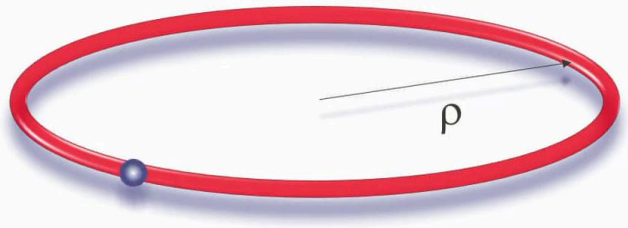
Notes

Summary

0m 05s

Centripetal forces

- F = ma
- Centripetal acceleration
  - $a = v^2/\rho \approx c^2/\rho$
    (N.B. also $a = \omega^2\rho$, $\omega = 2\pi\nu$)
  - Relativistic mass: $m_e \Rightarrow \gamma m_e$
- $\Rightarrow F_c = \gamma m_e c^2/\rho$



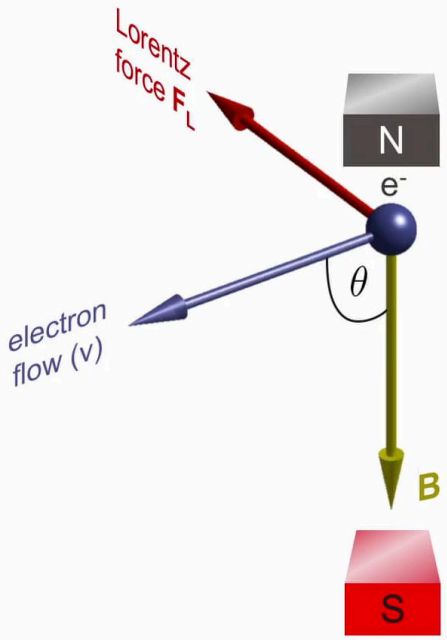
We begin by deriving a simple expression for the centripetal force needed to guide a relativistic electron in an arc with a radius rho. We all remember from school physics, or at least we should do, that the force is simply the mass times the acceleration. In the case of centripetal acceleration, this is equal to v squared divided by rho, or alternatively, if we use the angular frequency omega, which is equal to 2Pi mu, then this is equal to omega squared rho. But the velocity v is to a very high degree of accuracy, equal to the speed of light c. Moreover, we need to take into account that in the laboratory frame of reference, the electron appears to have a greater relativistic mass, which we have already stated is gamma times larger than the electrons rest mass. Gamma is of the order of a few thousand. Therefore, the expression for the centripetal force is simply Fc is equal to gamma me c squared divided by rho.

Notes

Summary

0m 44s

The Lorentz force again



$$\mathbf{F}_L = e\,\mathbf{B} \times \mathbf{v}$$
$$= e\,|B|\,|v|$$

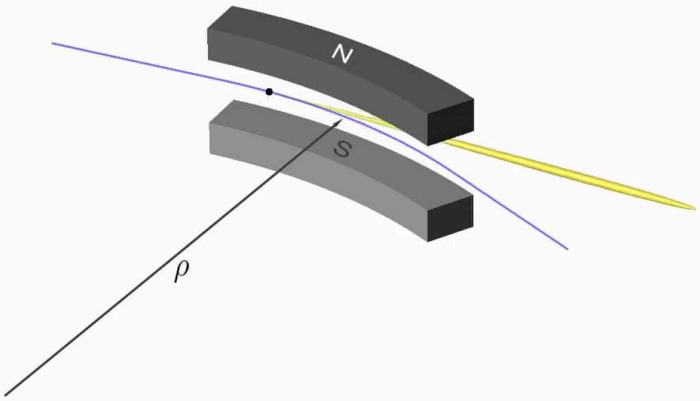
We now return to the Lorentz force imparted by a magnetic field of strength [inaudible 00:02:13] on a relativistically moving electron. We take the simplest case whereby the angle between the magnetic field and the electron velocity is 90 degrees, in which case we obtain the Lorenz force FL is equal to e B v.

Notes

Summary

2m 07s

Equating F_L and F_c



- $$\frac{\gamma m_e c^2}{\rho} = Bev \approx Bec$$

$$\Rightarrow \rho = \frac{\gamma m_e c}{Be} = \frac{\mathcal{E}}{Bec}$$

- $$\rho[\mathrm{m}] = 3.3 \frac{\mathcal{E}\,[\mathrm{GeV}]}{B\,[\mathrm{T}]}$$

We equate the centripetal and Lorentz forces to directly obtain an expression for the arc radius, namely rho is equal to gamma me c divided by Be. But gamma mec squared is the electron's energy in the storage ring, hence we can reformulate our expression for the bending radius to rho is equal to E divided by B little ec. Little e and c are both physical constants. So we can derive a simple practical expression for rho such that in units of metres, it's equal to 3.3 multiplied by the storage ring energy in giga electron volts divided by the magnetic field strength of the magnetic dipole given in tesla.

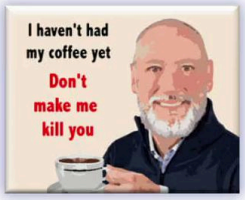
Notes

Summary

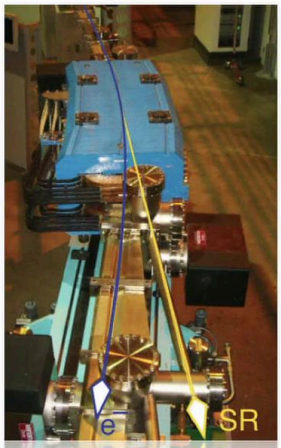
2m 33s

Typical magnetic field strengths

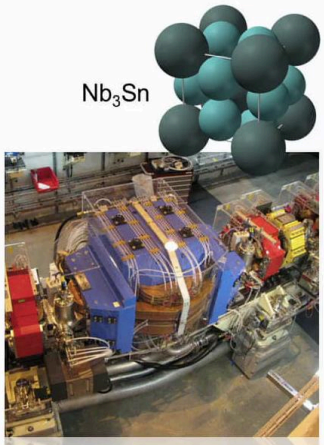
Earth's magnetic field
< 60 μT



Kitchen magnet
~ 30 mT



Bending magnet
~ 1 - 2 T



Cryogenic superbend
2 – 8 T

Now how strong are magnetic fields? The strongest values of the Earth's magnetic field close to the poles are a little less than 60 microtesla. Your common old garden kitchen magnet has a field strength nearly 1,000 times greater than this. Permanent or electric magnets used as synchrotrons to construct bending magnet dipoles are typically between about 1 and 2 tesla. Special materials such as niobium 310 or niobium titanium that need to be cryogenically cooled in order to work are utilised in so-called superbends, which are employed at lower energy storage rings to access higher photon energies than would otherwise be accessible. Note also that the forces between two magnets used at a synchrotron can be extremely large, equating to gravitational forces associated with masses of hundreds of kilograms concentrated across areas of perhaps only a few or few tens of square centimetres. Handling such permanent magnets, therefore, requires extremely stringent safety measures.

Notes

Summary

3m 29s

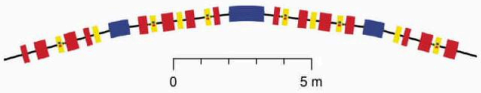
Typical bending magnet radii

$$\rho[\mathrm{m}] = 3.3 \frac{\mathcal{E}\,[\mathrm{GeV}]}{B\,[\mathrm{T}]}$$

$\rho \sim 10\,\mathrm{m}$ (normal BM)

$\rho \sim 1 - 2\,\mathrm{m}$ (superbend)

- Example: SLS, $\mathcal{E}$ = 2.4 GeV
  - BM: B = 1.4 T
  - ρ = 5.66 m
  - 2πρ = 35.54 m
  - SLS circumference = 288 m
  - 252 m of SLS are straight
    - Lengths between dipole magnets in triple-bend achromat arcs
      - ca. 170 m (12 x 14 m)



    - 12 straight sectors used for IDs
      - ca. 80 m

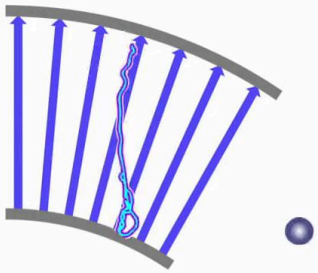
So modern magnetic materials at ambient temperature can provide magnetic field strengths of the order of one or two tesla. If we insert these values into our practical expression for the bending radius rho and insert a storage ring energy of 3 giga electron volts, we obtain values for rho of the order of 10 metres, a perfectly reasonable engineering size. Superbends have field strengths approximately 5 times greater, leading to bending radii of the order of 1 to 2 metres. But hang on a sec, synchrotrons have circumferences measured in hundreds of metres. So if we take the Swiss Light Source as an example, we obtain a bending radius of 5.66 metres for the 1.4 tesla magnets, which are used for the bendings. The circumference associated with this radius 2Pi rho is only 35 metres. That's over 250 metres less than the SLS circumference. Now, circumference here is perhaps a poor expression associated as this word is with a circle, as the footprint of any storage ring is actually not circular, but consists of lots of straight sections measured in several metres length reserved for insertion devices, plus arc sectors, which themselves are split into individual bending magnets, separated by short straights that accommodate other magnet elements such as quadrupoles and sextapoles, which we briefly discussed in the last video of this section.

Notes

Summary

4m 51s

How about using E-fields to deflect electrons?



- E-field, F = -eE = $\gamma m_e c^2/\rho$
  - ρ ~ 10 m
  - γ ~ 5000
  - m_e = 9.1 x 10^{-31} kg
  - c = 3 x 10^8 m/s
  - e = 1.6 x 10^{-19} C
  - $\Rightarrow$ E ~ 10^9 V/m
    - Far too high!! – electrostatic breakdown of matter
- Using electric fields to steer electrons in storage rings is impossible!!

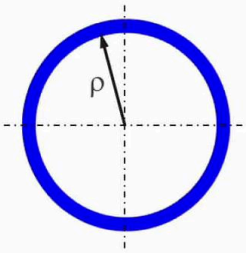
Now, just a brief side note before we continue. Why don't we manipulate the electron path using electron fields, for example? We can proceed as before in our calculations for magnetic field strengths, but now equating the centripetal force with the electric force minus eE, little e, big E. Inserting known values for the physical constants m, E, c, and little e, and using reasonable values for gamma and rho, we arrive at electric field strengths of around a billion volts per metre, both technologically impossible to achieve and of course far too high for normal material to tolerate, resulting in electrostatic breakdown. Moreover, the electric force is not a lazy force like the Lorenz force. If the electron finds itself moving not exactly perpendicularly to the electric field lines, the electric field will either accelerate or decelerate the electrons, which will result in increased storage ring electron energy dispersion. In other words, we can completely forget electric fields as a source for bending electrons in a storage ring.

Notes

Summary

6m 39s

Angular frequency of electrons



- Consider only curved paths at dipole bending magnets
- Circumference = $2\pi\rho$
- Speed ≃ c
- $\Rightarrow$ rotations/s = $c/2\pi\rho$
- $\Rightarrow$ ω_0 = radians/s = $2\pi c/2\pi\rho = c/\rho$

$$\omega_0 = c/\rho \sim 3 \times 10^7 \,\mathrm{rad\,s^{-1}}$$

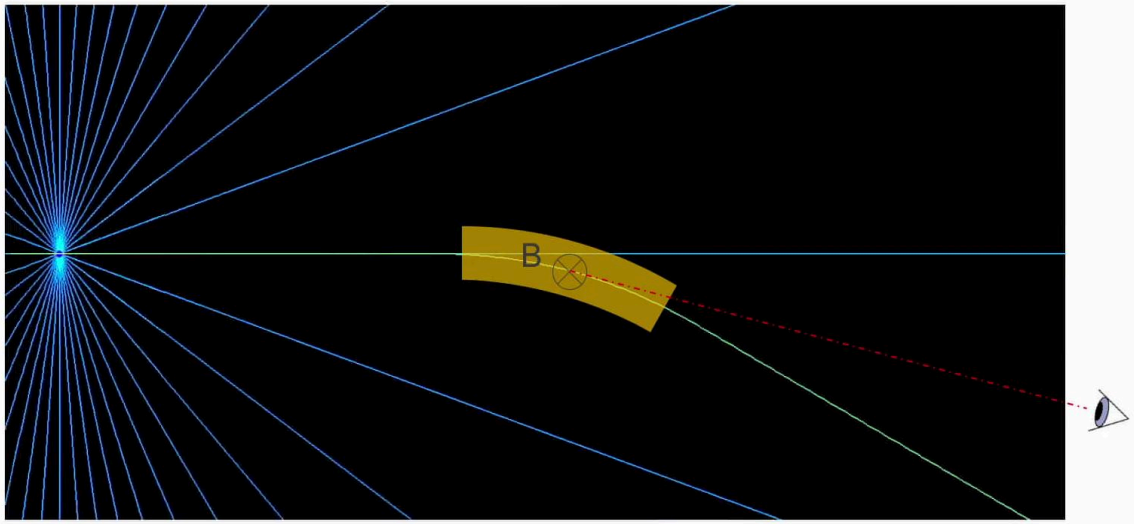
We now want to work out the angular frequency of the electrons, that is the speed in radians per second at which they negotiated the curved sections of their path. This is an important parameter in determining the spectrum emitted by bending magnets. We obviously need to ignore all the straight parts of the storage ring and only consider the arcs with radius rho. The sum length of the arcs is obviously 2Pi rho. And to all intents and purposes, we can again equate the electron velocity with c, the speed of light. Therefore, while the electrons are executing curved paths, they do this with c divided by 2Pi rho rotations per second. But a single full rotation contains 2Pi radians, hence the angular frequency in radians per second is simply omega 0 is equal to c divided by rho, which is of the order of 3 times 10 to the 7 radians per second. Keep this number in mind as it is a crucial parameter in determining the bending magnet spectrum.

Notes

Summary

8m 05s

Electric field generated by an electron traversing a BM



http://www.shintakelab.com/en/enEducationalSoft.htm

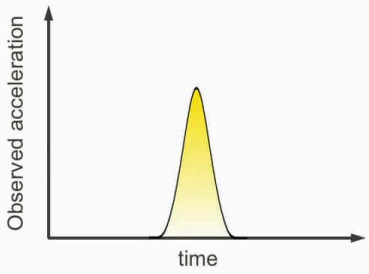
Before we quantify this, let's take a look at how the electric field appears to an observer looking at light emitted tangentially from a bending magnet. The observer sees a single lateral jerk in the electric field lines. As we already discussed last week, an electric field propagating to an observer with a non-zero perpendicular component to that propagation direction equates to the emission of electromagnetic radiation. If this lateral electric field component oscillates sinusoidally, the radiation will be monochromatic. Here, however, this is clearly not the case.

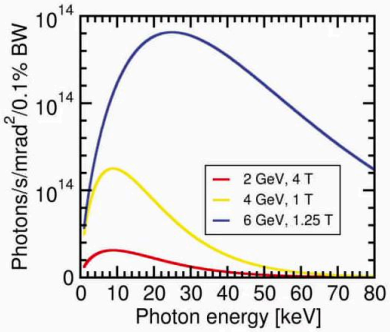
Notes

Summary

9m 30s

Bending magnet spectra





- Electrons experience centripetal acceleration towards bending arc centre
  - Produces a “pulse” of transverse electric-field component
  - Contrast with sinusoidal acceleration of Hertzian dipole radiation
- BM spectra cannot be described by a single or narrow bandwidth of frequencies
- BM spectra are broadband
  - Fourier transform of acceleration profile

See also a more detailed description of bending-magnet spectra in the supplementary text “Bending magnet spectra”

So what can we expect from this single centrifugal acceleration event? The observed pulse of transverse electric field is not dissimilar to a Gaussian pulse. The Fourier transform of this pulse represents the photon spectrum, as the Fourier transform of a Gaussian is also a Gaussian. Therefore, the bending magnet spectrum will also not be a million miles away from being Gaussian and will have a large relative bandwidth delta mu divided by mu. And indeed, this is the case, as can be seen in the graph here, for three selected combinations of storage, ring energy and magnetic field strength. It lies well beyond the scope of this course to go through the very detailed mathematical derivation of bending magnet spectra well beyond. However, a marginally more detailed description of this can be found in the supplementary text bending magnetic spectrum.

Notes

Summary

10m 20s

Critical frequency and energy of SR

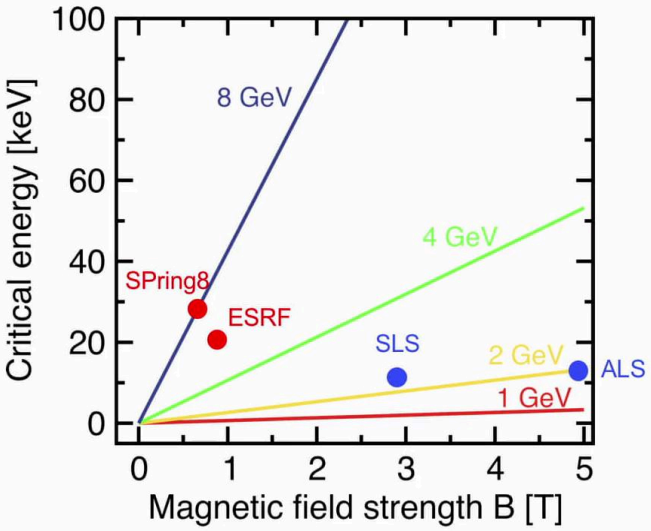
$$\omega_c = \frac{3}{2}\gamma^3\omega_0$$

$$\omega_c = \frac{3}{2}\gamma^3\frac{c}{\rho} = \frac{3}{2}\gamma^2\frac{eB}{m_e}$$

$$\hbar\omega_c[\mathrm{keV}] = 0.665\,\mathcal{E}^2\,[\mathrm{GeV}]\,B\,[\mathrm{T}]$$

Half of the total power emitted by a bending magnet lies below the critical (or “characteristic”) energy, half above

$$\frac{\hbar\omega_{\mathrm{max\,flux}}}{\hbar\omega_c} = \frac{1}{\sqrt{\ln 2}}$$



Where is the maximum of the bending magnetic spectrum? In order to determine this, we use the angular frequency omega 0. Don't forget, omega zero refers to the radians per second swept out by the electrons. The critical or characteristic frequency omega c is associated with the emitted radiation and is equal to 3/2 gamma cubed omega 0. But we know that rho is equal to gamma me c divided by B, and hence we can re-express omega c as 3/2 gamma squared eB divided by me. The critical energy is simply hPi times the critical angular frequency. In practical units, this is 0.665 E squared in giga electron volts times B in tesla, the energy being expressed in kilo electron volts. Note that half the emitted power from a bending magnet lies below the critical energy half above. Note also that although close, the critical energy is not at the maximum flux of the spectrum, but in fact lies a factor, 1 upon the square root of the natural logarithm of 2, which is equal to 1.201 higher than the maximum. On the right is a plot of the change in critical energy as a function of dipole field strength for different storage ring energies. Note that the gradient of each line is proportional to the square of the storage ring energy.

Notes

Summary

11m 28s

Critical frequency and energy of SR

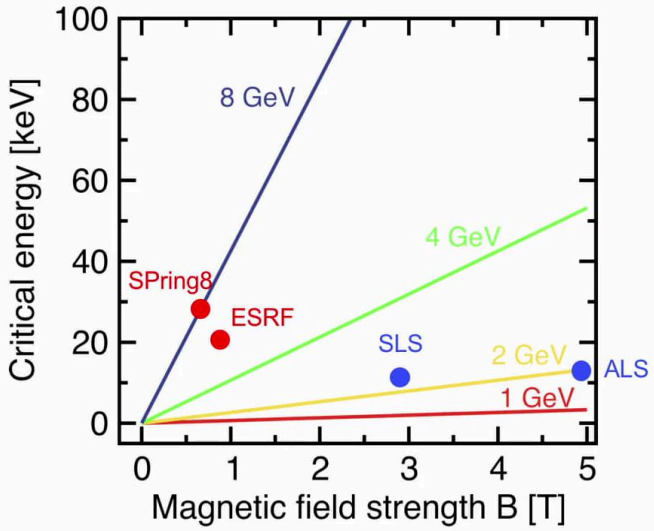
$$\omega_c = \frac{3}{2}\gamma^3\omega_0$$

$$\omega_c = \frac{3}{2}\gamma^3\frac{c}{\rho} = \frac{3}{2}\gamma^2\frac{eB}{m_e}$$

$$\hbar\omega_c[\mathrm{keV}] = 0.665\,\mathcal{E}^2\,[\mathrm{GeV}]\,B\,[\mathrm{T}]$$

Half of the total power emitted by a bending magnet lies below the critical (or “characteristic”) energy, half above

$$\frac{\hbar\omega_{\mathrm{max\,flux}}}{\hbar\omega_c} = \frac{1}{\sqrt{\ln 2}}$$



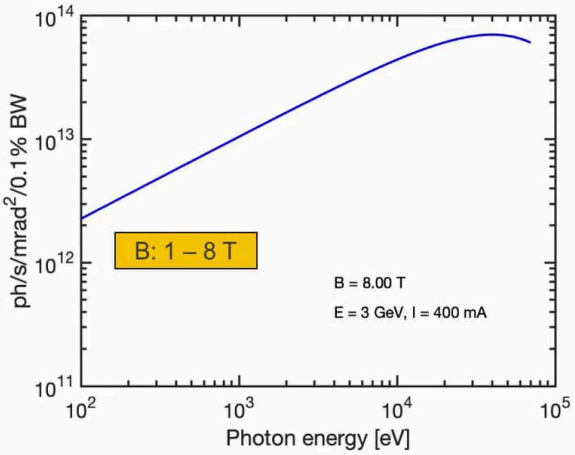
The two red circles are values of the critical energy for the bending magnets at the high energy facilities of SPring8 and ESRF. The two blue circles represent superbends at the medium energy facilities, SLS and the ALS.

Notes

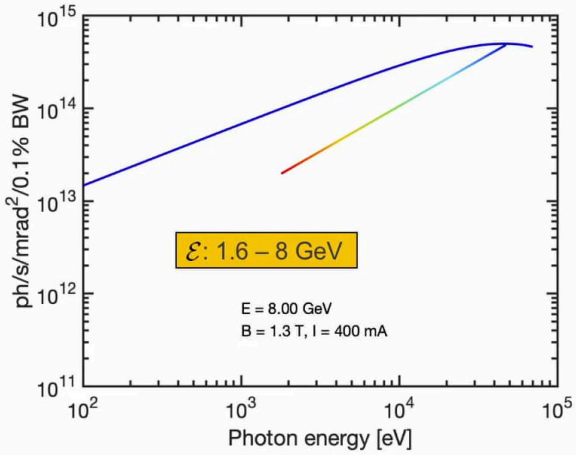
Summary

13m 21s

Bending magnet spectra



Peak flux independent of B
Peak photon energy proportional to B



Peak flux proportional to γ^2
Peak photon energy proportional to γ^2

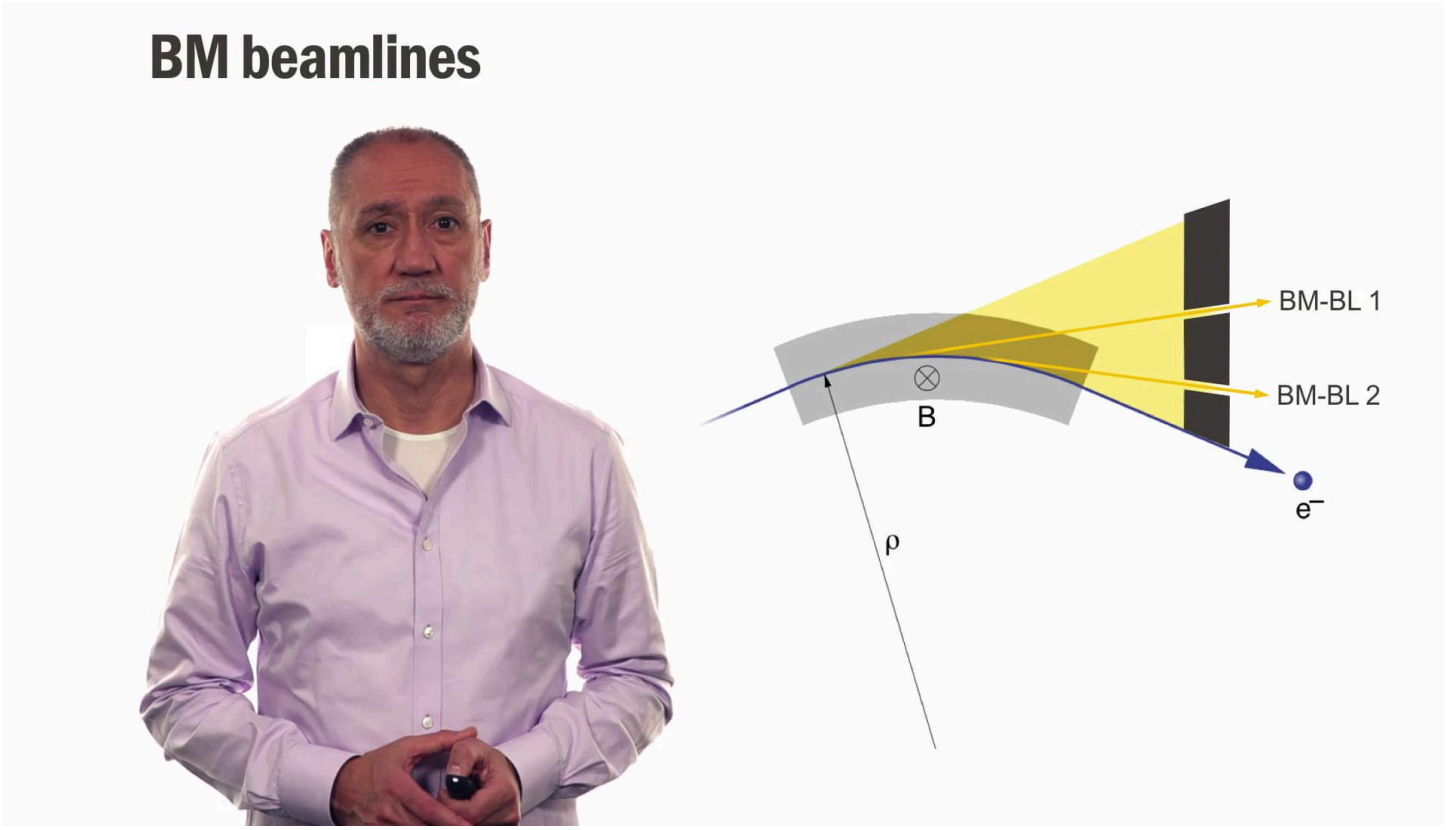
See also a more detailed description of bending-magnet spectra in the supplementary text "Bending magnet spectra"

Finally, the two animations here show how bending magnet spectra change with magnetic field strength and with storage ring energy. Note that the maximum flux does not change by increasing B alone only the position of the maximum, which is directly proportional to B. In contrast, the flux maximum increases as the square of the storage ring energy, while the flux also increases with this square of E.

Notes

Summary

13m 39s



Any given bending arc sector covers several degrees. As a consequence, more than one bending magnet beamline can be installed at a given arc and indeed many synchrotrons take advantage of this.

Notes

Summary

14m 13s

In the next video, we consider the heart of a synchrotron, namely the RF source. The electrons emit energy in the form of electromagnetic radiation, and without some energy source to replenish this, they will rapidly lose enough energy that they spiral towards the centre of the ring and are lost by drifting outside a stable orbit. As we will see, the RF supply stops this from happening, and, in the process, also makes the electron punch into discrete packets known as buckets.

Notes

Summary

14m 31s